

1. Determine if the following are linear mappings:

$$a) f: \mathbb{R}^2 \rightarrow \mathbb{R}^3, f \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ x_1 + x_2 \\ 5x_1 \end{pmatrix}.$$

$$b) f: \mathbb{R}^3 \rightarrow \mathbb{R}^2, f \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_1 + x_2 + x_3 \end{pmatrix}.$$

$$c) f: \mathbb{R}^3 \rightarrow \mathbb{R}^2, f \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

$$d) f: \mathbb{P}_2 \rightarrow \mathbb{P}_1, f(a_0 + a_1t + a_2t^2) = 2a_0 + a_1 + (a_2 - a_1)t.$$

$$e) f: \mathbb{R}^2 \rightarrow \mathbb{R}^2, f \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1^2 + x_2^2 \end{pmatrix}.$$

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$$f) f: M_{n,n} \rightarrow M_{n,n}, f(A) = A + A^t$$

$M_{n,n}$: vector space of square matrices
 $n \times n$

2. Consider the subset M of 2×2 square matrices defined as:

$$M = \left\{ \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} \text{ con } a, b, c \in \mathbb{R} \right\}$$

and the mapping $M \rightarrow M$ defined by:

$$f \left(\begin{pmatrix} a & b \\ c & 0 \end{pmatrix} \right) = \begin{pmatrix} a-b & b-a \\ c & 0 \end{pmatrix}.$$

Observe that M is a vector space and consider the basis

$$B = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$$

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Show that f is linear and compute its associated matrix with respect to B .

3. Consider the linear mapping $f: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ that maps the vectors

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

to the vectors

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

respectively. These vectors are coordinate vectors with respect to the canonical bases of \mathbb{R}^3 and \mathbb{R}^2 . Compute the associated matrix

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4. Define the linear mapping $f: V \rightarrow W$, where $\dim V = 3$ and $\dim W = 4$, such that $f(\bar{b}_1 - \bar{b}_3) = \bar{c}_1$, $f(\bar{b}_2 - \bar{b}_3) = \bar{c}_1 - \bar{c}_2$, and $f(2\bar{b}_3) = 2\bar{c}_1 + 2\bar{c}_3$. Here, $B = \{\bar{b}_1, \bar{b}_2, \bar{b}_3\}$ is a basis of V and $C = \{\bar{c}_1, \bar{c}_2, \bar{c}_3, \bar{c}_4\}$ is a basis of W .

a) Find the associated matrix with respect to B and C .

b) Find a basis for $\text{Im} f$

c) Find a basis for $\text{Ker} f$

The logo for Cartagena99 features the text 'Cartagena99' in a stylized, dark green font. The '99' is significantly larger and more prominent than the 'Cartagena' part. The text is set against a light blue background with a white swoosh underneath.

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5. Determine if the following mappings:

a) $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $f \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 \\ x_1 + x_2 \\ -x_3 \end{pmatrix},$

b) $f: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $f \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ 0 \end{pmatrix},$

are linear mappings. If they are linear, find their kernel and range. Study their injectivity and surjectivity.

The logo for Cartagena99 features the text 'Cartagena99' in a stylized, bold font. The '99' is significantly larger and more prominent than the 'Cartagena' part. The text is set against a background of overlapping light blue and orange geometric shapes, possibly representing a globe or abstract design.

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6. Determine whether the following statements are true or false. Justify your answer.

a) If $g: V \rightarrow W$ is a linear mapping, sometimes it is possible to find three distinct vectors $\bar{u}, \bar{v} \in V$ and $\bar{w} \in W$ such that

$$g(\bar{u}) = g(\bar{v}) = \bar{w}.$$

b) Assuming that the previous statement is true, if $g(\bar{u}) = g(\bar{v}) = \bar{w}$, then $\bar{u} - \bar{v} \in \ker g$.

c) If $g: V \rightarrow W$ is a linear mapping, then the range of g is W .

d) If $\{\bar{v}_1, \bar{v}_2, \dots, \bar{v}_n\}$ is a basis of \mathbb{R}^n and $\{\bar{w}_1, \bar{w}_2, \dots, \bar{w}_n\}$ is a basis of \mathbb{P}_{n-1} , then there are two linear mappings

$f: \mathbb{R}^n \rightarrow \mathbb{P}_{n-1}$ and $g: \mathbb{P}_{n-1} \rightarrow \mathbb{R}^n$ such that

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e) If $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear mapping defined by $f(\vec{0}) = \vec{0}$, then f is identically the null mapping ($f(\vec{x}) = \vec{0} \quad \forall \vec{x} \in \mathbb{R}^2$).

f) There is a linear mapping $f: \mathbb{R}^5 \rightarrow \mathbb{R}^5$ with $\dim \ker f = \dim \operatorname{Im} f$.

g) Assuming that $f: M_{2,2} \rightarrow M_{2,2}$ is linear with $\dim \operatorname{Im} f = 4$, if $f(A) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, then $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

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7. Define the linear mapping $f:V \rightarrow W$ where $\dim V = \dim W = 3$, such that $f(\bar{e}_1) = \bar{u}_1 - \bar{u}_2$, $f(\bar{e}_2) = \bar{u}_2$, and $f(\bar{e}_3) = \bar{u}_1$. Here, $B = \{\bar{e}_1, \bar{e}_2, \bar{e}_3\}$ is a basis of V and $C = \{\bar{u}_1, \bar{u}_2, \bar{u}_3\}$ is a basis of W .

a) Find the associated matrix with respect to B y C .

b) Find $\dim \text{Im} f$.

c) Find a basis of $\text{Ker} f$.

d) Given a new basis of V , $\tilde{B} = \{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ where $\bar{e}_1 = \bar{v}_1$, $\bar{e}_2 = \bar{v}_1 + \bar{v}_2$, and $\bar{e}_3 = \bar{v}_1 + \bar{v}_3$, compute the associated matrix with respect to \tilde{B} and C .

e) Given the following change of basis

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$$[\bar{w}]_{\tilde{C}} = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} [\bar{w}]_C, \quad \bar{w} \in W$$

compute the associated matrix with respect to \mathcal{B} and $\tilde{\mathcal{C}}$.

g) Obtain the matrix associated with f with respect to $\tilde{\mathcal{B}}$ and $\tilde{\mathcal{C}}$.

h) study the injectivity and surjectivity of f

8. In \mathbb{R}^3 , consider the basis $\{\bar{e}_1, \bar{e}_2, \bar{e}_3\}$. Study the injectivity and surjectivity of the linear mapping $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $f(\bar{e}_1) = a\bar{e}_1 + \bar{e}_2 + \bar{e}_3$, $f(\bar{e}_2) = \bar{e}_1 + \bar{e}_2 + \bar{e}_3$, and

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